

SERGO TSIRAMUA

STRUCTURAL ANALYSIS OF COMPLEX SYSTEMS

Academic-Practical Course



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The textbook "Structural Analysis of Complex Systems" was developed within the framework of the 2022 internal grant project of The University of Georgia, titled "Development of a Computational Software Suite for Structural Analysis of Systems Using Logical-Probabilistic Methods", which was carried out under the supervision of the author of this book.

To automate the learning process and enable the completion of practical tasks on a computer, accompanying software was developed for the textbook, available at www.ssa.ug.edu.ge. The web application is based on the logical-probabilistic methods, models, and algorithms discussed in the master's course "Structural Analysis of Systems." The author extends sincere gratitude to all colleagues and students who participated in the project.

With the use of this textbook and its software, graduate and doctoral students at the School of Science and Technology will be able to perform mathematical modeling of the design and operation of complex systems, quantitatively evaluate system efficiency criteria, and solve problems related to optimal control.

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Introduction

The textbook discusses the analysis and operations of different types of complex systems, their components (elements), structures, and processes. The basis of the systematic approach, theory of the systems and concepts, characterization, and principles are discussed. The textbook displays the criteria of efficiency of complex systems, their reliability, viability, fault tolerance, flexibility, maneuverability, risks, and safety. Using the methods of the analysis of structural systems shows that the effectiveness of any given system thoroughly depends on its structure – on the arrangement of the elements and their interconnection. It is precisely due to the structure that a high-level system of reliability can be achieved with the elements with low-level reliability, and similarly, on the contrary, a high level of system reliability cannot be achieved solely with high-reliability elements [1]. Therefore, in order to design a highly reliable system with an optimal structure, it is essential to conduct their structural analysis, research, and optimal design process. This should be based on the earlier stages of modeling the system's operational process and the quantitative assessment of efficiency criteria.

The textbook bases the complex system's research, structural analysis modeling of the operational process and the quantitative assessment of efficiency indicators on universal logical-probabilistic methods and can be applied to a broader range of systems.

The main principle of the logical-probabilistic methods is the following: The system's structure and functionality are described by the logical functions. By transforming logical functions, an algebraic expression is obtained where logical variables are replaced by their probabilistic indicators, and logical operators are substituted with mathematical operations. The incorporation of probabilistic data in the given expression enables the quantitative, specifically probabilistic, assessment of the system's efficiency indicators. The study and optimization of the system structures are based on the quantitative assessment indicators of system efficiency criteria, which is crucial for the synthesis and design of high-efficiency systems.

To assess the risks and the safety indicators of the system, the method of describing the scenario of a system transitioning into a hazardous (dangerous) state using a logical function is used. The logical elements of this function also represent hazard-initiating events and initiating conditions. The logical function of the scenario of the hazardous state can also be reduced to a probabilistic model, which enables the probabilistic assessment of risks and safety. This helps to identify the vulnerabilities of the system and for optimizing future risks.

The textbook discusses logical-probabilistic modeling not only for single-functional elements but also for reconfigurable and adaptable structured systems based on multi-functional elements, which is said to be less studied and cover the broader scope of research.

The textbook will cover the topics related to the description of structures and processes in technical, computational, informational, and human-machine systems, the development of principal and logical schemes, the quantitative assessment of efficiency criteria, as well as the analysis and optimization (improvement, enhancement) of system structures. In order to improve the learning process, the practical assignments will be performed on the computer program Logisim.exe and the program made specifically for the “Structural Analysis of Systems” Masters course – www.ssa.ug.edu.ge.

The textbook is intended for the master's and doctoral students specializing in informatics. IT management, business, engineering, and mathematics, as well as for specialists interested in these topics.

Chapter 1

Systems and their Assessment Criteria

1.1. The System Theory and Systemic Approach

While researching the technical, informative, economic, ecological, financial, political, social, and other kinds of events or processes, it is important to have a systematic approach, which is based on the interaction of systems with their environment, the study of system elements (components, objects), their interconnections (structure) and the analysis of the external and internal processes. Naturally, while studying the efficiency of any system, network, or complex – including reliability, safety, and risks – it is important to consider a systemic approach.

Famous Georgian Scientist Iveri Phrangishvili, while considering the latest advancements in system theory, defines that the main scientific significance of the systemic approach is that it enables modern scientists to identify and study systematic principles, which is evident in nearly every event and process in nature, society and individuals. The systemic approach is based on a shared vision of the objects, events, and processes. It serves as the most universal and adequate method for analyzing and researching the technical, economic, social, ecological, political, and other types of systems. Therefore, a systemic approach in management enables the identification of the essence and content of management mechanisms, as well as the exploration of new management concepts. The proof of this statement became possible thanks to the scientists of the end of the last century and the beginning of the 21st century who developed and used powerful tools such as system and process modeling. Without these tools, it would be impossible to identify and study not only the systemic features of different types of objects but also manage any process – whether it be a scientific process, governance, or management at any level, from companies to individual subjects [2].

System theory entails the interactions of processes and their influence on each other over a specific time period, allowing for the continuity of a larger whole.

System theory is a transdisciplinary study of systems, meaning groups of interconnected, interdependent components, which may be natural or human-made. A systemic approach is a form of applying knowledge and dialect to study the processes occurring in nature, society, and thought. Systemic approach directs researcher's attention toward revealing the integrity of an object, identifying the diversity of connections within it and a unified theoretical framework.

Systemic approach is the most universal method of studying and analyzing complex systems. Objects are examined as systems consisting of regularly structured and functionally organized elements. A systemic approach is the systematization and integration of objects, or knowledge about them, by establishing essential connections between them. The systemic approach entails a sequential transition from general to specific, where the basis of the analysis is a specific final goal for which the given system is created. This approach implies that each system functions as an integrated entity even when it consists of separate and distinct subsystems.

The essence of the systemic approach lies in completing the requirements of general system theory, where each object should be examined in the process of studying it as both a large and complex system and, at the same time, as an element of a broader system. Comparatively independent components

are discussed in interconnection as opposed to in isolation. Once a system's single component changes, others change as well. This makes it possible to identify a system's integration properties and qualitative characteristics that do not exist within the constituent elements of a system.

Various tools are used to convey the essence of a system: Graphical, Mathematical, Matrix, "Decision tree," and so on. By using Computer technologies based on a systemic approach, it is possible to improve management methods and structures.

The key concepts of the systematic approach are: "**system**", "**structure**" and "**component**."

"System" – A set of components (elements, objects) that are interconnected, whose interaction generates a new quality that is not inherent in these components individually.

"Component" – Considered as any object that is complexly connected to other objects.

"Structure" – Defined as the scheme of element arrangement and the distribution of functions among elements within the system. It involves the arrangement of systems and the nature of the interaction of properties. The structure connects, transforms elements, gives a certain wholeness and leads to the emergence of new properties that are not inherently present in any individual component. An object is considered a system if it can be divided into interconnected and interacting components. These parts, in turn, usually have their own structure and, therefore, are represented as subsystems of the original, larger system.

The main principles of systemic approach are:

Wholeness, which allows a system to be considered simultaneously as a whole and, at the same time, as a subsystem of a higher-level system.

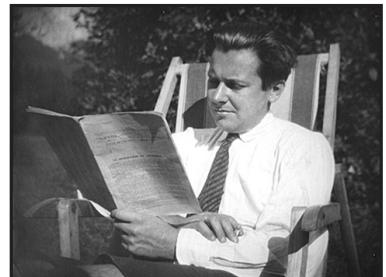
Structure hierarchy, meaning the existence of a set of elements (minimum 2), which is based on the subordination of lower-level elements to higher-level elements.

Structuring, which allows for the analysis of system elements and their interrelationships within a specific organizational structure. Usually, the process of functioning of a system is determined not by the properties of its individual elements but by the characteristics of the structure itself.

Multiplicity, which allows for the application of various cybernetic, economic, and mathematical models to describe both individual elements and the system as a whole.

Systemic approach considers the unique unity of a system with its environment. It is defined as a set of external elements that influence the interaction of the system's components.

Systemic approach as a general methodological principle is applied across various scientific disciplines and areas of human activity. Its epistemological foundation (epistemology being a branch of philosophy that studies the forms and methods of scientific knowledge) is the general theory of systems, founded by the Austrian biologist **Ludwig von Bertalanffy**.



Karl Ludwig von Bertalanffy (September 19, 1901 – June 12, 1972) was an Austrian biologist known as one of the founders of General Systems Theory (GST). GST is an interdisciplinary approach that describes systems as interacting components and is applied in biology, cybernetics, and various other fields.

He saw the goal of this science as the search for structural similarities in the laws established across different disciplines, based on which it would be possible to derive large-scale system patterns. Today, many scientific articles are dedicated to systemic research; they all share a commitment to solving systemic problems, where the research object is viewed as a system. The broadest interpretation of the systemic approach methodology belongs to Professor Ludwig von Bertalanffy, who first introduced the idea of “General Systems Theory” as early as 1937. The subject of “General Systems Theory” is defined by Bertalanffy as the importance of establishing general principles that apply generally for systems. “The existence of common properties in systems,” he wrote, “results in the manifestation of structural similarities, or isomorphism, across different fields.” This is due to the fact that these components, at some extent, can be considered as “systems”-complexes of elements that interact with each other. In reality, similar concepts, models, and laws often appear independently and describe and research entirely different phenomena.

The first characteristic of the systemic approach is that it is a form of methodological knowledge applied exclusively to systems that are related to the study and formation (creation) of the objects as systems.

The second characteristic of the systemic approach is the hierarchy of knowledge, which requires a multilevel study of a subject: the “own” level – studying the subject itself; The “higher” level – examining the same subject as an element of a broader system; The “lower” level – analyzing the subject in relation to its constituent elements.

The third characteristic of the systemic approach is the study of integrational properties of systems and complexes, their patterns, and the understanding of the key integration mechanisms.

The fourth most important characteristic of systemic approach is its focus on obtaining quantitative characteristics, developing methods that clarify the ambiguity of concepts, definitions and assessments. In other words, the systemic approach requires considering the problem not in isolation but in interaction with its environment, understanding the essence of each connection and individual element, and establishing associations between general and specific objectives. All of this creates a unique method of thinking, enabling more agility to situational changes and facilitating informed decision-making. Consequently, we define the concept of a systemic approach.

The concept of systemic approach is a method of studying an object (problem, phenomenon, or process) as a system, where elements, internal and external relationships are identified – those that have the most significant impact on its functioning outcomes and the goals of each element. This is determined based on the overall purpose of the object.

In practice, in order to implement a systemic approach, it is important to ensure following sequence of actions:

- Formulation of the research problem;
- Identification of the research object as a system;
- Establishment of the system’s internal structure and identification of external components;
- Definition (or setting) of element goals based on the identified (or expected) outcome of the overall system;
- Development of a system model and conducting research on it.

There can be two types of systemic tasks: System analysis or system synthesis.

The task of analysis involves determining the properties of a system based on its own structure, whereas the task of synthesis focuses on defining the system's structure based on its desired properties.

The goal of synthesis is to create a new structure with the required characteristics, while the goal of analysis is to study the properties of an already existing formation.

Thus, the systemic approach is a methodological scientific discipline with the goal of developing methods for studying and constructing systems of various types and classes.

You may come across a dual understanding of the systemic approach: on the one hand, it involves the study and analysis of existing systems, and on the other hand, it focuses on the creation and synthesis of systems to achieve specific goals.

System analysis is used as one of the most essential methods in systemic approach as the effective tool for researching complex, normal or vaguely defined problems.

Systems engineering is an applied science that studies the problems of creating complex management systems.

The process of building a management system consists of six stages:

1. System analysis.
2. System planning, which includes defining current objectives, scheduling, and work plans.
3. System design – the actual design of a system, its subsystems, and components to achieve optimal efficiency.
4. Software development.
5. System implementation and testing.
6. System maintenance.

The study and classification of systems are related to determining the “input-output” relationship – how influence (input) affects outcome (output). The input is supplied to the system's entry point, while the result is recorded as its output. A system model consists of four fundamental elements: “Input,” “Process,” “Output,” and “Feedback.” The process consists of operations that transform the input into the desired output. The input represents the core resource, which is converted into the output (see Fig.1.1) [2].

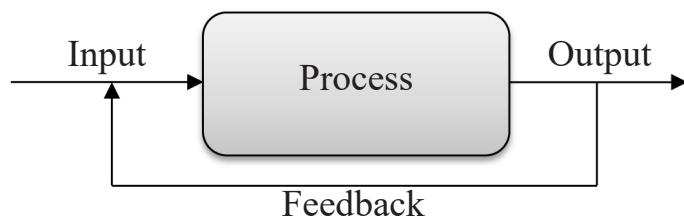


Fig. 1.1 “Input – Output”

The degree of system organization is typically expressed through the synergy effect. This means that the overall performance of the system exceeds the sum of the individual performances of its constituent elements. In practice, this means that the same set of elements can form systems with either identical or different properties but with varying levels of efficiency, depending on how the elements are interconnected and how functions are distributed among them – in other words, how the system is organized [2].

1.2. System reliability and safety

The development of system theory in science has a history of over a half-century, yet there is still no universally accepted definition of a system. This diversity arises from the fact that in different interpretations of the concept of a system, its philosophical, physical, mathematical, linguistic, and other aspects are emphasized. According to “ganmartaebi.ge Georgian dictionary”, a system is defined as follows [3]:

1. A wholeness composed of interconnected parts that follow a specific pattern (e.g., the planetary system, the nervous system).
2. A set of principles belonging to a scientific, political, or other doctrine.
3. A method or set of rules systematically applied to carry out a specific task (e.g., the electoral system, education system).
4. A form of social organization, such as a democratic system, capitalist system, or authoritarian governance system.
5. A network of institutions that are organizationally unified (e.g., institutions with medical systems).
6. A technical device or construction.

Despite a variety of definitions, we have to discuss any system as a multitude of elements whose interconnections form a single unified whole.

A “**system**” is a set of interconnected elements considered as a single structural whole. These interconnections (relationships) among elements distinguish a system from a mere conglomerate of its constituent parts.

A **system** (Ancient Greek: σύστημα – «correspondence») – refers to a certain order based on the structured arrangement and interconnection of its parts.

A system can be:

- A set of principles forming the foundation of a specific doctrine.
- A grouping or classification (e.g., Linnaeus' botanical system, the periodic table of chemical elements).
- A structure or a whole consisting of interconnecting parts that follow a specific pattern (e.g., the scholar system, nervous system, neural network, or computer network).

System classes. There are natural and artificial, open and closed, simple and complex systems, organizational, technical, informational, humanitarian, aggregate, and others. We can also discuss energetic, industrial, transportation, computing, automated systems, etc.

The textbook primarily focuses on studying complex structured systems, including technical, informational, computational, and human-machine systems. However, for comparative analysis, simple structured systems are also discussed.

As mentioned above, a system consists of functionally interconnected elements that together form the structure of the system.

“**Structure**” is defined as the arrangement and functional interconnections of the components (elements, objects or parts) that make up a system.

The term **structure** (Latin: Structūra – framework, arrangement, order) refers to the set of stable relationships within an object (system) that ensures its integrity, identity, and the preservation of its essential properties despite various external and internal changes.

A **process** (Latin: Processus – progress, advancement) is a sequence of actions performed to achieve a specific outcome. For example, a production process consists of the sequential execution of labor operations, and a computer program executes a series of commands (algorithms) in a computer.

To describe a process means to algorithmize it. Algorithmizing a process involves describing the process in text form, schematic representation, mathematical symbols, or algorithmic languages. The result is a process algorithm representing the elementary actions (tasks, operations), their sequence, and interconnections.

Complex structured systems are systems whose structural description does not align with classical sequential, parallel, or tree-like structures.

Any class of complex structured systems is described using a network-type scenario, for which **logical-probabilistic methods** are applied.

One of the methods for analyzing, studying, and designing complex structured systems is computer-based **logical-probabilistic modeling**.

In the design and operation of complex structured systems, it is essential to ensure high reliability, safety, and minimal risks. This is the primary objective of our course, and we will address these challenges using structural system analysis methods and logical-probabilistic modeling.

The issues of reliability and safety have always accompanied scientific and technological progress. As a scientific discipline, reliability theory emerged in the mid-20th century, coinciding with the rapid development of missile complexes, nuclear power plants, digital computing technology, and computer networks. Over the past half-century, numerous studies and publications have been dedicated to forming reliability theory, enhancing reliability, and developing quantitative assessment methods for reliability criteria. It is worth mentioning that these research efforts and developed methodologies have led to the creation of high-reliability systems in recent decades. However, reliability remains an ongoing challenge, and the refinement of methodologies remains an active area of research.

When studying system reliability and safety, it is advisable to introduce definitions of efficiency criteria to avoid conceptual confusion. Reliability encompasses a set of criteria, including faultlessness, fault tolerance, and viability.

In general, system **reliability** can be defined as the ability of a system to maintain the necessary properties to perform its designated function within a given time interval.

A fundamental concept in reliability theory is **faultlessness** – the ability of a system to maintain its operational capability without failures over a specified period under normal operating conditions.

“System failure” is an event in which a system loses its ability to perform its assigned function, meaning it becomes inoperative. In most cases, this occurs due to the failure of one or more of its components (elements).

The faultlessness of a technical (computational) system is its ability to maintain all parameter values that define its capability to perform assigned functions within a given time interval under specified operating conditions and modes, including during maintenance, repair, storage, and transportation.

Fault tolerance (fault-resistant capability) is a system's ability to maintain operational functionality even in the event of component (element, subsystem) failures, using mechanisms such as duplication, redundancy, adaptation, and automatic recovery.

In general, fault tolerance is a system's ability to continue functioning even in the event of one or more component failures.

Functional survivability is a system's ability to retain the necessary properties required to perform its assigned function, even at reduced efficiency, under adverse external influences and force majeure situations that are not accounted for under normal operating conditions (e.g., explosions, fires, earthquakes, floods, etc.).

System safety differs from reliability – a reliable system is not necessarily a safe system, and vice versa [4, 5, 6, 8].

Definitions of safety:

A dangerous condition is a state that may lead to hazard, disaster, catastrophe, or a collapse, potentially putting people or objects at risk, causing significant damage, and even resulting in human casualties.

Danger is a potential event or occurrence that may pose a threat to someone or something, potentially leading to the system's transition into a dangerous condition.

Safety is a state where no danger is present, and all necessary measures are in place to prevent potential threats. System safety is the ability of a system to perform its assigned function without transitioning into a dangerous condition or by effectively mitigating risks.

An initiating event is a hazard-triggering occurrence that may cause the system to transition into a dangerous condition.

An initiating condition is an event that may fail to prevent a hazard and instead facilitate the initiating event, leading the system into a dangerous condition [4, 5, 6].

In the following chapters of the textbook, you will explore the description of complex structure systems and their operational conditions using Boolean algebra. Additionally, you will learn about logical-probabilistic methods for quantitatively assessing system reliability and safety.

Assignment 1

Theoretical questions:

1. What is systems theory, and what does it study?
2. What is the systemic approach, what is its essence, and how is it represented?
3. What is the fundamental principle of the systemic approach?
4. List and briefly describe the key concepts of the systemic approach.
5. List and briefly describe the characteristics of the systemic approach.
6. What is the concept of the systemic approach?
7. What actions are necessary for the practical implementation of the systemic approach?
8. Describe the two types of systemic tasks – “system analysis” and “system synthesis.”
9. List the stages of the process of building a management system.
10. What is a system, element, structure, and process?
11. List the different classes of systems.
12. Define the reliability criteria of a system: reliability, failure, faultlessness, fault tolerance, and survivability.
13. Define the safety criteria of a system: dangerous condition, danger, safety, initiating event, and initiating condition.

Practical assignments:

14. Provide three examples of non-technical systems (different from those described in the textbook).
15. Provide three examples of technical systems.
16. Provide three examples of computer-based computational systems.